XIV INTERNATIONAL SYMPOSIUM ON

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BOOK OF ABSTRACTS

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Contents

Keynote Speakers

The Boosted Double-Proximal Subgradient Algorithm for Nonconvex Optimization

Francisco J. Aragón-Artacho * [1] , **Pedro Pérez-Aros** [2] , **David Torregrosa-Belén** [3]

Abstract

In this talk we present a new splitting algorithm that can be used to tackle very general structured nonconvex minimization problems of the form

$$\min_{x \in \mathbb{R}^n} f(x) + g(x) - \sum_{i=1}^{p} h_i(\Psi_i(x)), \qquad (\mathcal{P})$$

where the function $f : \mathbb{R}^n \to \mathbb{R}$ is locally Lipschitz and satisfies the descent lemma, $g : \mathbb{R}^n \to]-\infty, +\infty]$ is lower-semicontinuous and prox-bounded, $h_i : \mathbb{R}^{m_i} \to \mathbb{R}$ are convex continuous functions and $\Psi_i : \mathbb{R}^n \to \mathbb{R}^{m_i}$ are differentiable functions with Lipschitz continuous gradients, for $i = 1, \ldots, p$. Our algorithm makes use of subgradients of f, the gradients of the differentiable functions Ψ_i, and proximal steps of the functions g and of the conjugates of the convex functions h_i. As, in addition, it includes a line-search step that allows to enhance its performance, we name it Boosted Double-proximal Subgradient Algorithm (BDSA). While BDSA encompasses existing schemes proposed in the literature, it extends its applicability to more diverse problem domains. We present some convergence results and conclude with some numerical experiments evaluating the performance of BDSA.

References

(1) Aragón-Artacho, F.J., Pérez-Aros, P., Torregrosa-Belén, D., The Boosted Double-proximal Subgradient Algorithm for nonconvex optimization. Submitted (2024), arXiv: 2306.17144.

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New results on the multi-dimensional linear discriminant analysis problem

Amir Beck [1]

Abstract

Fisher linear discriminant analysis (LDA) is a well-known technique for dimensionality reduction and classification. The method was first formulated in 1936 by Fisher. We present three different formulations of the multi-dimensional problem and show why two of them are equivalent. We then prove a rate of convergence of the form $O(q^{k^2})$ for solving the third model. Finally, we consider the max-min LDA problem in which the objective function seeks to maximize the minimum separation among all distances between all classes. We show how this problem can be reduced into a quadratic programming problem with nonconvex polyhedral constraints and describe an effective branch and bound method for its solution.
Joint work with Raz Sharon.

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A new algorithm for the problem of maximizing the difference of two convex functions

Aharon Ben-Tal * [1] , **Luba Tetruashvili** [2]

Abstract

Maximizing the difference of 2 convex functions over a convex feasible set (the so called DCA problem) is a hard problem. There is very large number of publications for solving this problem. Most of them are variations of the widely used DCA algorithm. The success of these algorithms for reaching a good approximation of a global solution, depends crucially on the choice of its a starting point. We introduce a new algorithm (NDC) which is notoriously different from DCA. A major effort effort in NDC is the generation of a good starting point. This is based on the COMAX algorithm for maximizing globally constrained convex problem (A. Ben-Tal, E. Roos, INFORMS J. ON COMPUTING,Vol.34, Issue 6).

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A review on strongly quasiconvex functions: existence, characterizations and algorithms

Felipe Lara * [1]

Abstract

In this talk, we provide a review on the class of strongly quasiconvex functions introduced by Polyak in 1966 (6). First, we focus on the existence of solutions for the minimization problem of strongly quasiconvex functions and its consequences in the properties for the proximity operator. Second, we provide first-and second-order characterizations for differentiable strongly quasiconvex functions via the behaviour of its gradient by extending the famous characterization of Arrow and Enthoven given in (1) and, as a consequence, we present a new generalized monotonicity which is exactly between strongly monotone and strongly pseudomonotone operators. Third, we provide new and finer optimality conditions for optimization problems defined with strongly quasiconvex functions which improves the well-known optimality conditions for the quasiconvex case. Fourth, we apply the previous results for establishing exponential convergence for the first- and second-order gradient systems and linear convergence rate for their explicit discretizations which leads to the gradient descent method and the heavy ball method, respectively. Finally, and if the time allow us, we study the proximal gradient method for the sum of two nonconvex functions in which one of them is nonsmooth and strongly quasiconvex while the other is differentiable with Lipschitz continuous gradient and possible nonconvex too.

References

(1) Arrow, K.J., Enthoven, A.C. (1961), Quasiconcave programming, *Econometrica*, vol.29, pp.779–800. ISSN 0012-9682. DOI: 10.2307/1911819

(2) Kabgani, A., Lara, F. (2022), Strong subdifferentials: theory and applications in nonconvex optimization, *J. of Global Optim.*, vol.84, pp.349–368. ISSN: 0925-5001, DOI: 10.1007/s10898-022-01149-9

(3) Lara F. (2022), On strongly quasiconvex functions: existence results and proximal point algorithms, *J. Optim. Theory Appl.*, vol.192, pp.891–911. ISSN 0022-3239, DOI: 10.1007/s10957-021-01996-8

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(4) Lara F., Marcavillaca, R.T., Vuong, P.T. (2024), Characterizations, dynamical systems and gradient methods for strongly quasiconvex functions, *Submitted*.

(5) Lara F., Yen, L.H. (2024), Further information on the minimization of the sum of nonconvex functions with applications to mathematical programming, *Submitted*.

(6) Polyak, B.T. (1966), Existence theorems and convergence of minimizing sequences in extremum problems with restrictions, *Soviet Math.*, vol.7, pp.72–75.

Generalized Differentiation and Applications to Optimization: from Convexity to Nonconvexity

Nguyen Mau Nam [1] ,

Abstract

In this presentation, we introduce our recent advances in developing calculus rules of generalized differentiation for nonsmooth functions and set-valued mappings in both finite and infinite dimensions, spanning from convexity to nonconvexity. Our approach includes developing new results involving convex separation using the generalized relative interior, generalized relative interiors of graphs of convex and nearly convex set-valued mappings, the introduction of a novel concept of Fenchel conjugate set-valued mappings, and the utilization of a variational geometric approach to provide a unified method for obtaining and significantly extending many existing results of convex analysis. We also explore how generalized convexity can address nonconvex optimization challenges, particularly those that are of a nonsmooth nature. Our focus is on studying multifacility location and clustering problems from both theoretical and numerical perspectives. We introduce optimization techniques that involve Nesterov's smoothings, exponential smoothing, and minimizing differences of convex functions to design algorithms for solving new models of facility location and clustering.

References

(1) An, N.T., Nam, N.M., Yen, N.D. (2016), A DC algorithm via convex analysis approach for solving a location problem involving sets, Journal of Convex Analysis, vol. 23, pp. 77–101. ISSN 0944-6532.

(2) An, N.T., Nam, N.M., Qin, X. (2020), Solving k-center problems involving sets based on optimization techniques, Journal of Global Optimization, vol. 76, pp. 189–209. ISSN 0925-5001. DOI: 10.1007/s10898-019-00858-2.

(3) Bajaj, A., Mordukhovich, B., Nam, N.M., Tran, T. (2022), Solving a continuous multifacility location problem by DC algorithms, Optimization Methods and Software, vol. 37, pp. 338–360. ISSN 1055-6788. DOI: 10.1080/10556788.2021.1871619.

(4) Cuong, D.V., Mordukhovich, B., Nam, N.M. (2021), Quasi-relative interiors for graphs of convex set-valued mappings, Optimization Letters, vol. 15, pp. 933–952. ISSN 1862-4472. DOI: 10.1007/s11590-020-01621-5.

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(5) Long, V.S.T., Mordukhovich, B., Nam, N.M. (2023), Generalized relative interiors and generalized convexity in infinite dimensions, submitted to Optimization.

(6) Luan, N.N., Nam, N.M., Thieu, N.N. (2023), Relationships Between Polyhedral Convex Sets and Generalized Polyhedral Convex Sets, Journal of Optimization Theory and Applications, vol. 199, pp. 766–786. ISSN 0022-3239. DOI: 10.1007/s10957-023-02269-2.

(7) Luan, N.N., Nam, N.M., Yen, N.D. (2024), Properties of generalized polyhedral convex multifunctions, submitted to Journal of Set-Valued and Variational Analysis.

(8) Mordukhovich, B., Nam, N.M. (2022), Convex Analysis and Beyond, Vol. 1: Basic Theory, Springer Series in Operations Research and Financial Engineering, Springer. ISBN 978-3-030-60053-6.

(9) Mordukhovich, B., Nam, N.M. (2023), An Easy Path to Convex Analysis and Applications: Second Edition, Synthesis Lectures Series on Mathematics and Statistics, Springer. ISBN 978-3-030-60053-6.

(10) Nam, N.M., Geremew, W., Reynolds, S., Tran, T. (2018), The Nesterov smoothing technique and minimizing differences of convex functions for hierarchical clustering, Optimization Letters, vol. 12, pp. 455–473. ISSN 1862-4472. DOI: 10.1007/s11590-017-1191-1.

(11) Nam, N.M., Sandine, G., Thieu, N.N., Yen, N.D. (2024), Fenchel conjugate of set-valued mappings, submitted to Journal of Optimization Theory and Applications.

(12) Nam, N.M., Thieu, N.N., Yen, N.D. (2024), Near convexity and generalized differentiation, Journal of Convex Analysis, forthcoming.

Coderivative-Based Semi-Newton Method in Nonsmooth Programming

Pedro Pérez-Aros[1] ,

Abstract

This presentation introduces a novel Newton-type algorithm designed for a class of nonsmooth optimization problems, where the objective is defined as the difference between two nonconvex functions. The algorithm is based on the coderivative-generated second-order subdifferential and employs advanced tools of variational analysis. Well-posedness properties of the proposed algorithm are derived under fairly general requirements, while constructive convergence rates are established by incorporating additional assumptions, including the Kurdyka–Łojasiewicz condition. Finally, numerical experiments showcase the versatility of our algorithm.

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Stochastic oracles and where to find them

Katya Scheinberg [1]

Abstract

Continuous optimization is a mature field, which has recently undergone major expansion and change. One of the key new directions is the development of methods that do not require exact information about the objective function. Nevertheless, the majority of these methods, from stochastic gradient descent to "zero-th order" methods use some kind of approximate first order information. We will overview different methods of obtaining this information, including simple stochastic gradient via sampling, robust gradient estimation in adversarial settings, traditional and randomized finite difference methods and more. We will discuss what key properties of these inexact, stochastic first order oracles are useful for convergence analysis of optimization methods that use them.

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Monday, September 2

Construction of Vector-Valued Weak Separation Functions with Applications to Conjugate Duality in Vector Optimization

Christiane Tammer * [1] , **Chaoli Yao** [2] , **Siqi Wang** [3]

Abstract

In our previous paper (1), collections of scalar weak separation functions for image space analysis were proposed, while, this talk is concerned with vector-valued weak separation functions. Applying vector-valued nonlinear weak separation and topical functions (see (2)), a framework of conjugate duality for constrained vector optimization problems is established. Using vector-valued separation functions, the dual model is given for the vector optimization problem directly, unlike the previous work (3), where the primal problem was scalarized. In our new approach to duality, we avoid a scalarization in the formulation of the dual problem. We study a pair of a primal vector-valued problem and a dual set-valued problem. Simultaneously, the zero duality gap and strong duality are studied by certain concepts of subdifferentials, separation and saddle points in the vector-valued sense.

References

(1) Yao, C.L. and C. Tammer (2023), Weak separation functions constructed by Gerstewitz and topical functions with applications in conjugate duality, *J. Nonlinear Var. Anal.*, vol. 7, n. 5, pp.859-896.

(2) Rubinov, A.M. and I. Singer (2001), Topical and sub-topical functions, downward sets and abstract convexity, *Optimization*, vol.50, n.5-6, pp.307-351.

(3) Yao, C.L. and S.J. Li (2020), Conjugate duality for constrained vector optimization in abstract convex frame. *Numer. Func. Anal. Optim.*, vol.40, pp. 1242-1267.

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Nonlinear Cone Separation Theorems and Applications in Optimization

Christian Günther * [1] , **Bahareh Khazayel** [2] , **Christiane Tammer** [3]

Abstract In this talk, we present some new results for the separation of two not necessarily convex cones by a (convex) cone / conical surface in real (reflexive) normed spaces. These results have been published in (3) and (4). Our approach follows the nonlinear and nonsymmetric separation approach developed by Kasimbeyli (1), which is based on augmented dual cones and Bishop-Phelps type separating functions. In comparison to Kasimbeyli's separation theorem, our theorems for the separation of two cones are formulated under weaker conditions (concerning convexity and closedness requirements) with respect to the involved cones. A novel characterization of the algebraic interior of augmented dual cones in real normed spaces enables the establishment of relationships between our cone separation results and those derived by Kasimbeyli (1) and by Garcia-Castano, Melguizo-Padial, and Parzanese (2). We also highlight some applications in optimization.

References

(1) Kasimbeyli, R. (2010), A nonlinear cone separation theorem and scalarization in nonconvex vector optimization, *SIAM J Optim*, vol. 20, n. 3, pp. 1591–1619. DOI: 10.1137/070694089.

(2) Garcia-Castano F., Melguizo-Padial M.A., Parzanese G. (2023), Sublinear scalarizations for proper and approximate proper efficient points in nonconvex vector optimization, *Math Meth Oper Res*, vol. 97, pp. 367–382. DOI: 10.1007/s00186-023-00818-z.

(3) Günther C., Khazayel B., Tammer C. (2023), Nonlinear cone separation theorems in real topological-linear spaces, *SIAM J Optim*, vol. 34, nr. 1, pp. 225–250. DOI: 10.1137/22M1542003.

(4) Günther C., Khazayel B., Tammer C. (2024), Nonlinear strict cone separation theorems in real normed spaces, *J Nonlinear Var Anal*, accepted. DOI preprint: 10.48550/arXiv.2303.06392.

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Generalized Conic Scalarization in Vector Optimization

Bahareh Khazayel * [1] , **Christian Günther** [2] , **Christiane Tammer** [3]

Abstract It is well known that scalarization techniques (e.g., in the sense of Gerstewitz; Kasimbeyli; Pascoletti and Serafini; Zaffaroni) are useful for finding efficient solutions of vector optimization problems. One recognized approach is the conic scalarization method proposed by Kasimbeyli (1), which is based on augmented dual cones and Bishop-Phelps type scalarization functions. In this talk, we present (from the paper (5)) a generalized version of the conic scalarization method and new scalarization results for different types of proper efficiency concepts (e.g., in the sense of Benson; Borwein; Henig). To derive our new results, we use some recent strict cone separation theorems by Günther, Khazayel and Tammer (3,4) and recent scalarization results by Jahn (2).

References

(1) Kasimbeyli R. (2010), A nonlinear cone separation theorem and scalarization in nonconvex vector optimization, *SIAM J Optim*, vol. 20, n. 3, pp. 1591–1619. DOI: 10.1137/070694089.

(2) Jahn J. (2023), A unified approach to Bishop-Phelps and scalarizing functionals, *J Appl Numer Optim*, vol. 5, n. 1, pp. 5–25. DOI: 10.23952/jano.5.2023.1.02

(3) Günther C., Khazayel B., Tammer C. (2023), Nonlinear cone separation theorems in real topological-linear spaces, *SIAM J Optim*, vol. 34, nr. 1, pp. 225–250. DOI: 10.1137/22M1542003.

(4) Günther C., Khazayel B., Tammer C. (2024), Nonlinear strict cone separation theorems in real normed spaces, *J Nonlinear Var Anal*, accepted. DOI preprint: 10.48550/arXiv.2303.06392.

(5) Günther C., Khazayel B., Tammer C. (2024), On generalized conic scalarization in vector optimization in real topological-linear spaces, working paper.

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On conic non-smooth semi-infinite program having vanishing constraints

Tamanna Yadav [1] , **S. K. Gupta** * [2] , **Sumit Kumar** [3]

Abstract

A mathematical program with infinite constraints is known as a semi-infinite problem. The focus of this talk is on a specific class of optimization problems known as non-smooth conic semi-infinite programming problems having constraints that vanish under certain conditions. This formulation encompass a wide range of existing models due to the inclusion of arbitrary cones. Using the limiting constraint qualification and the (basic) subdifferentials, a necessary optimality condition for this semi-infinite optimization model is established. Further, using the concept of generalized convexity over cones, the sufficient optimality conditions are also established. Moreover, two dual models namely Wolfe's and Mond-Weir's types are formulated for the considered semi-infinite optimization problem. By assuming generalized Q-convexity/pseudoconvexity/Q-quasiconvexity, weak, strong, and converse duality results are further derived. These results shed light on the relationships between primal and dual problems under different convexity assumptions. Various non-trivial examples are also constructed to validate the derived results.

References

(1) Mordukhovich, B.S. (2018), Variational analysis and applications, (Vol. 30), *Springer-Cham.*

(2) Hu, Q., Wang, J., & Chen, Y. (2020), New dualities for mathematical programs with vanishing constraints, *Annals of Operations Research*, Vol.287, pp.233–255. ISSN 1572-9338. DOI: 10.1007/s10479-019-03409-6.

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A variational approach to weakly continuous relations in Banach spaces

Didier Aussel [1] **Massimiliano Giuli** [2] , **Monica Milasi** * [3] , **Domenico Scopelliti** [4]

Abstract

Optimization and equilibrium problems have been extensively studied when the involved preference relations admit a representation by means of real-valued functions. Although these problems have been analyzed under very minimal assumptions on the representation function, this context could appear to be quite restrictive in some practical situations.

By using tools of variational analysis and by weakening some continuity properties, we aim to provide, in an infinite dimensional setting, an alternative approach to deal with the study of preference relations without (necessarily) representation means of real-valued functions. As an application of our theoretical developments, we analyze a particular preference equilibrium problem by using a suitable quasi-variational inequality formulation.

References

(1) Aussel D., Giuli M., Milasi M., Scopelliti D., A variational approach to weakly continuous relations in Banach spaces, has to appear.

(2) Aussel, D. and Hadjisavvas, N. (2005), Adjusted sublevel sets, normal operator, and quasi-convex programming, *SIAM Journal of Optimization*, vol. 16, pag. 358-367.

(3) Castellani M., Giuli M. (2024), A continuity result for the adjusted normal cone operator, *Journal of Optimization Theory and Applications*, vol. 200, pp. 858–873.

(4) Milasi M., Scopelliti D. (2021), A variational approach to the maximization of preferences without numerical representation, *Journal of Optimization Theory and Applications*, vol. 190, pp. 879-893.

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(5) Kreps, D. (2013), Microeconomics fundations I: choice and competitive markets, *Princeton University Press.*

Integrability of the subdifferential of a p.l.n. function on a Banach space

Milen Ivanov [1] , **Matey Konstantinov** [2] , **Nadia Zlateva** [3]

Abstract

The *primal lower nice (p.l.n.)* functions are defined by Poliquin in (1). Building upon the method developed in (2) and extending it nontrivially we prove the integrability of p.l.n. functions on a Banach spaces.

References

(1) R. A. Poliquin, *Integration of subdifferentials of nonconvex functions*, Nonlinear Anal. Th. Meth. Appl., 17 (1991), 385–398

(2) M. Ivanov and N. Zlateva, *Slopes and Moreau-Rockafellar Theorem*, arXiv:2309.03505, 2023.

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On Convexity of Images of Sets under Nonlinear Mappings

Y.S. Ledyaev [1] , **A. Uderzo** * [2] ,

Abstract

We consider infinitesimal conditions for a nonlinear mapping $F : X \longrightarrow Y$ which provide strict convexity of the image $F(C)$ of a closed set $C \subset X$. The predecessor of these results is a celebrated Convexity Principle due to B.T. Polyak, who demonstrated that, under assumptions of surjectivty and local Lipschitz continuity of derivative F', images of small balls are convex. This result was then generalized in numerous papers (e.g. by Ivanov, Ledyaev, Uderzo) but they all assumed strict convexity of the preimage set C. Our current results are obtained in the more general case of C only convex or even non-convex. They are based on a generalization of famous Klee's theorem on locally convex sets for the case of locally strictly convex sets.

References

(1) Ivanov, G.E. (2020), Nonlinear Images of Sets. I: Strong and Weak Convexity, *J. Convex Anal.*, vol. 27, n. 1, pp. 363–382.

(2) Ledyaev, Yu.S. (2019), Criteria for the convexity of closed sets in Banach spaces, *Proc. Steklov Inst. Math.*, vol. 304, n. 1, pp. 190–204.

(3) Polyak, B.T. (2001), Convexity of nonlinear image of a small ball with applications to optimization, *Set-Valued Anal.* vol. 9 , no. 1-2, pp. 159–168.

(4) Uderzo, A. (2013), On the Polyak convexity principle and its application to variational analysis, *Nonlinear Anal.*, vol. 91, pp. 60-71.

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A penalty barrier framework for constrained optimization

Alberto De Marchi * [1] , **Andreas Themelis** [2]

Abstract

Minimization problems with structured objective function (sum of smooth and prox-friendly terms) and smooth (equality and inequality) constraints provide a broad template, covering models such as nonlinear, equilibrium, sparse, and disjunctive programming: Focusing on this problem class, synopsized in the form

$$\text{minimize } f(x) + g(x) \qquad \text{over } x \in \mathbb{R}^n \qquad \text{subject to } l \leq c(x) \leq u,$$

we present a flexible technique that combines the beneficial regularization effects of (exact) penalty and interior-point methods. Working in the fully nonconvex setting, a pure barrier approach requires careful steps when approaching the infeasible set, thus hindering convergence [1]. We show how a tight integration with a penalty scheme allows us to avoid such conservatism, at the price of having an infeasible method. Although the iterates are allowed outside of the feasible set, feasibility is promoted via a penalty term while complementarity is controlled with the barrier parameter [2]. Thanks to this coupling, the combined method does not require a strictly feasible starting point, and thus accommodates equality constraints. The crucial advancement that allows us to invoke generic (possibly accelerated) subsolvers is a marginalization step: amounting to a conjugacy operation, this step effectively merges (exact) penalty and barrier into a smooth, full domain functional object. When the penalty exactness takes effect, e.g., under some regularity qualifications, the generated subproblems do not suffer the ill-conditioning typical of interior-point methods, nor do they exhibit the nonsmoothness of exact penalty terms. We provide a theoretical characterization of the algorithm and its asymptotic properties, deriving convergence results for fully nonconvex problems. If at least an accumulation point of the sequence generated by our algorithm is feasible, then it returns after finitely many iterations a primal-dual pair approximately satisfying suitable optimality conditions. Furthermore, we show that infeasible accumulation points are stationary for a suitable feasibility problem. Stronger results are available for the convex setting, where feasibility can be guaranteed [3]. Illustrative examples and numerical simulations demonstrate the wide range of problems our theory and algorithm are able to cover.

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References

(1) A. De Marchi and A. Themelis (2022), An interior proximal gradient method for nonconvex optimization, arXiv:2208.00799v2 [math.OC].

(2) F.E. Curtis (2012), A penalty-interior-point algorithm for nonlinear constrained optimization, *Mathematical Programming Computation*, vol.4, n.2, pp.181–209. DOI: 10.1007/s12532-012-0041-4.

(3) E. Chouzenoux, M.-C. Corbineau and J.-C. Pesquet (2020), A Proximal Interior Point Algorithm with Applications to Image Processing, *Journal of Mathematical Imaging and Vision*, vol.62, n.6, pp.919–940. DOI: 10.1007/s10851-019-00916-w.

Absolute Value Penalty Function Method for Isoperimetric Constrained Variational Control Problem involving Uncertainty

Anurag Jayswal * [1] , **Ayushi Baranwal** [2] ,

Abstract

This study aims to investigate robust optimality for an isoperimetric constrained variational control problem involving uncertainty. In order to do this, we first use an auxiliary variable to convert the isoperimetric constraints into a system of ordinary differential equations. Then, we establish the robust necessary optimality criteria for the problem under consideration and use the notion of convex functional to verify its sufficiency. Furthermore, using the absolute value penalty function method, we formulate an unconstrained penalized problem associated with the considered constrained variational control problem. Thereafter, various equivalence results have been proved between the robust minimizer to the unconstrained penalized problem and the robust optimal solution to the constrained variational problem. We have also given numerical examples to support the theoretical results.

References

(1) Antczak, T. (2012), The vector exact l_1 penalty method for non-differentiable convex multiobjective programming problems, *Applied Mathematics and Computation*, vol. 218, pp. 9095–9106. ISSN 0096-3003. DOI: 10.1016/j.amc.2012.02.056

(2) Ben-Tal, A. and Nemirovski, A. (1998), Robust convex optimization, *Mathematics of Operations Research*, vol.23, pp. 769-805. ISSN 0364-765X. DOI: 10.1287/moor.23.4.769

(3) Jayswal, A., Preeti and Jimenez, M. A. (2022), Robust penalty function method for an uncertain multi-time control optimization problems, *Journal of Mathematical Analysis and Applications*, vol.505, pp. 125453, ISSN 0022-247X. DOI: 10.1016/j.jmaa.2021.125453

(4) Treanta, S. (2022), Lagrange-hamilton approach in optimization problems with isoperimetric-type constraints, *Journal of Optimization Theory and Applications*, vol.194, pp. 408-520. ISSN 0022-3239. DOI: 10.1007/s10957-022-02036-9

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Tuesday, September 3

Solving a Class of Nonconvex Quadratic Programs by Inertial DC Algorithms

Tran Hung Cuong [1] , **Yongdo Lim** [2] , **Nguyen Nang Thieu** [3] , **Nguyen Dong Yen** * [4]

Abstract

Two inertial DC algorithms for indefinite quadratic programs under linear constraints (IQPs) are considered in this paper. Using a qualification condition related to the normal cones of unbounded pseudo-faces of the polyhedral convex constraint set, the recession cones of the corresponding faces, and the quadratic form describing the objective function, we prove that the iteration sequences in question are bounded if the given IQP has a finite optimal value. Any cluster point of such a sequence is a KKT point. The convergence of the members of a DCA sequence produced by one of the two inertial algorithms to just one connected component of the KKT point set is also obtained. To do so, we revisit the inertial algorithm for DC programming of de Oliveira and Tcheou [de Oliveira, W., Tcheou, M.P.: *An inertial algorithm for DC programming*, Set-Valued and Variational Analysis 2019; 27: 895–919] and give a refined version of Theorem 1 from that paper, which can be used for IQPs with unbounded constraint sets. An illustrative example is proposed.

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On proximal algorithms with inertial effects beyond monotonicity

Alfredo Iusem * [1] , **R. T. Marcavillaca** [2]

Abstract

Inertial procedures attached to classical methods for solving monotone inclusion and optimization problems, which arise from an implicit discretization of second-order differential equations, have shown a remarkable acceleration effect with respect to these classical algorithms. Among these classical methods, one can mention steepest descent, alternate directions, splitting algorithms and the proximal point method. For the problem of finding zeroes of operators, the convergence analysis of all existing inertial-proximal methods requires monotonicity of the operator. We present here a new inertial-proximal point algorithm for finding zeroes of operators, whose convergence is established for a relevant class of nonmonotone operators, namely the hypomonotone ones.

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Quasiconvexity and Single-Leader-Multi-Follower: a perfect wedding

Didier Aussel [1]

Abstract

Quasiconvexity and game theory have been linked for many decades. For example the important role of the quasiconcavity of utility functions in equilibrium problem is well-known. On the other hand, in the last decade, complex game problems known as Single-Leader-Multi-Follower games have attracted attention and theoretical and algorithmic progresses have been done.

Our aim in this talk is to present a serie of very recent works showing how quasiconvexity plays again an important role for Single-Leader-Multi-Follower games.

References

- E. Allevi, D. Aussel, R. Riccardi and D. Scopelitti, Single-Leader-Radner-Equilibrium: a new approach for a class of bilevel problems under uncertainty, *J. Optim. Theory Appl.*, 200 (2024), 344-370.

- D. Aussel, D. Salas, K. Cao Van, Existence results for generalized Nash equilibrium problems under continuity-like properties of sublevel sets., *SIAM J. Optim.* (2021), Vol. 31, No. 4, pp. 2784-2806.

- D. Aussel & A. Svensson, Some remarks on existence of equilibria, and the validity of the EPCC reformulation for multi-leader-follower games, *J. Nonlinear Convex Anal.*, 19 (2018), 1141-1162.

- D. Aussel and A. Svensson, Chapter *A short state of the art on Multi-Leader-Follower Games*, pp. 53-76, In: Dempe S., Zemkoho A. (eds) Bilevel Optimization. Springer Optimization and Its Applications, vol 161 (2021), Springer, Cham.

- D. Aussel, M. Guili, M. Milasi and D. Scopelliti, Preference Relations without Numerical Representation in Banach spaces, submitted (2023), 20 pp.

- D. Aussel and P. Chaipunya, Local analysis of variational and quasi-variational inequalities: concept, properties and some applications, submitted (2023), 20pp.

[1] Lab. PROMES UPR CNRS 8521, University of Perpignan via Domitia, France.

- D. Aussel, C. Egea and M. Schmidt, A Tutorial on Solving Single-Leader-Multi-Follower Problems using SOS1 Reformulations, submitted (2023), 24 pp.

- D. Aussel, V. Jeyakumar and G. Li, Reformulations of Single-Leader-Multi-Follower Games without Convexity of Inequality Constraints and Applications, submitted (2024), 20 pp.

A stochastic bilevel model for energy transition: an analysis of the contract for difference application

Giorgia Oggioni * [1] , **Ruth Domínguez** [2] ,
Rossana Riccardi [3] , **Carlos Ruiz** [4]

Abstract In this work, we propose a stochastic bilevel programming problem where a strategic producer optimally operates its wind farm and its battery installation in a target year and the market regulator auctions off contracts for differences (CfD) for wind technologies. In the upper-level model, the strategic producer maximizes its own profit by deciding how much wind capacity to destine to its participation in the CfD market and how much of this capacity to direct commit to the electricity spot market. The strategic wind producer also owns an energy storage (battery) connected to the wind farm that is used to counteract the variability of the wind power production. In the lower-level problem, the regulator clears the bids and assigns the wind capacity accepted to be covered by the CfD contracts to market participants. In doing that, it minimizes the costs of derivative contracts while trying to finance wind capacity expansion projects up to a certain level. The case study is applied to the Spanish electricity market and different sensitivity analyses on the size of the CfD auction size, on the battery investment costs, and on the reference prices are conducted.

References

(1) Andreis L., Flora M., Fontini F., Vargiolu T. (2020), Pricing reliability options under different electricity price regimes, *Energy Economics*, vol.87, 104705. ISSN 0140-9883. DOI: https://doi.org/10.1016/j.eneco.2020.104705.

(2) Beck Y., Ljubić I., Schmidt M. (2023), A survey on bilevel optimization under uncertainty, *European Journal of Operational Research*, vol. 311, n. 2, pp.401-426. ISSN 0377-2217. DOI: https://doi.org/10.1016/j.ejor.2023.01.008.

(3) Dempe S., Kalashnikov V., Pérez-Valdés G., Kalashnykova N. (2015), Bilevel Programming Problems - Theory, Algorithms and Applications to Energy Networks, *Springer*, ISSN: 1867-8998. DOI: 10.1007/978-3-662-45827-3.

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(4) Fabra N. (2023), Reforming European electricity markets: Lessons from the energy crisis, *Energy Economics*, vol. 126, 106963. ISSN 0140-9883. DOI: https://doi.org/10.1016/j.eneco.2023.106963.

(5) Schlecht I., Maurer C., Hirth L. (2024), Financial contracts for differences: The problems with conventional CfDs in electricity markets and how forward contracts can help solve them, *Energy Policy*, vol. 186, 113981. ISSN 0301-4215. DOI: https://doi.org/10.1016/j.enpol.2024.113981.

An improved bilevel programming approach for assortment and cut of defective stocks

Andrea Pizzuti * [1] , **Claudio Arbib** [2] , **Fabrizio Marinelli** [3]
Mustafa Ç. Pınar [1]

Abstract

We consider a real assortment-and-cut problem, arising in the glass industry, that calls for fulfilling some known demand of small rectangular items by cutting only a restricted variety of rectangular plates. The material to be cut is prone to defects that can reduce the quality of the product or even cause the loss of some items.

Two recent studies investigated approaches to minimize the impact of random defects that may hit the plates. One of these proposes a bilevel optimization model to maximize the expected faultless production, where defect occurrence is simulated by an adversary that can place at most one random defect in each plate. The leader's recourse to this action consists in modifying, when possible, the pattern layout so that the defect falls in the scrap area and the value of the items lost is minimized.

A good feature of this model is that it can be reduced to a single-level integer program. However, the approach is on one hand too conservative as the distribution of defects, following a drastic worst-case perspective, results too particular to be likely; on the other hand, it is too restrictive, as it postulates no more than one defect per plate.

We propose an improved bilevel programming formulation that preserves the desirable reduction property and outdoes the aforementioned limitations. Computational experiments on realistic instances are presented to highlight the benefits achieved by the improved approach and to study the viability of the method.

References

(1) Arbib C., Marinelli F. (2007), An optimization model for trim loss minimization in an automotive glass plant, *European J. of Operational Research*, vol.183, n.3, pp.1421-1432.
DOI: 10.1016/j.ejor.2005.07.034.

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(2) Arbib C., Marinelli F., Pinar M.Ç., Pizzuti A. (2022), Robust stock assortment and cutting under defects in automotive glass production, *Production and Operations Management*, vol.31, n.4, pp.4154-4172.
DOI: 10.1111/poms.13812.

(3) Arbib C., Marinelli F., Pinar M.Ç., Pizzuti A. (2022), Assortment and cut of defective stocks by bilevel programming, *Proceedings of the 11th International Conference on Operations Research and Enterprise Systems*, vol.1, pp.294-301.
DOI: 10.5220/0000155400003117.

Optimality conditions for differentiable linearly constrained pseudoconvex programs

Riccardo Cambini [1] , **Rossana Riccardi** [2] *

Abstract

The aim of this paper is to study optimality conditions for differentiable linearly constrained pseudoconvex programs. The stated results are based on new transversality conditions which can be used instead of complementarity ones. Necessary and sufficient optimality conditions are stated under suitable generalized convexity properties. Moreover, two different pairs of dual problems are proposed and weak and strong duality results proved. Finally, it is shown how transversality conditions can be applied to characterize optimality of convex quadratic problems and to efficiently solve a particular class of Max-Min problems.

References

(1) Bard J.F. (2010), Practical Bilevel Optimization: Algorithms and Applications, *Nonconvex Optimization and Its Applications*, vol.30, Springer US, ISBN: 978-1441948076, DO!: 10.1007/978-1-4757-2836-1.

(2) Cambini R., Riccardi R. and D. Scopelliti (2023), Solving linear multiplicative programs via branch-and-bound: a computational experience, *Computational Management Science*, DOI: 10.1007/s10287-023-00471-1.

(3) Geoffrion A.M. (1971), Duality in non-linear programming: a simplified applications-oriented development, *SIAM Review*, vol.13, n.1, pp.1-37. DOI: 10.1137/1013001.

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Rank-two programs: generalized convexity and solution methods

Riccardo Cambini * [1] , **Giovanna D'Inverno** [2] , **Puya Latafat** [3]

Abstract

In this talk we first point out how generalized convexity can be useful in studying general rank-two problems. The stated results allow to propose a solution method based on a branch and bound approach. Then, the rank-two problems are transformed in a bilevel-like form and approached by means of KKT conditions. In this light, two solution method are proposed to manage the complementarity conditions, that is either by means of dummy binary variables or by means of a transversality condition.

References

(1) Cambini R. (2020), Underestimation functions for a rank-two partitioning method, *Decisions in Economics and Finance*. DOI: 10.1007/s10203-020-00288-6.

(2) Cambini R. and I. Venturi (2021), A new solution method for a class of large dimension rank-two nonconvex programs, *IMA Journal of Management Mathematics*. DOI: 10.1093/imaman/dpaa001.

(3) Cambini R. and G. D'Inverno (2023), Rank-two programs involving linear fractional functions, *Decisions in Economics and Finance*. DOI: 10.1007/s10203-024-00444-2.

(4) Cambini R. and R. Riccardi (submitted), Optimality conditions for differentiable linearly constrained pseudoconvex programs.

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Projected solutions for quasiequilibrium problems: theory and algorithm

M. Castellani * [1] , **G. Bigi** [2] , **S. Latini** [3]

Abstract

The concept of a projected solution for quasivariational inequalities and generalized Nash equilibrium problems was first introduced in [1]. This concept was motivated by the study of a pay-as-bid model for a deregulated electricity market. This work aims to show that the projected solutions of a quasiequilibrium problem correspond to the classical solutions of an auxiliary quasiequilibrium problem. This can be achieved by doubling the number of variables and adding an appropriate term. The auxiliary problem can be solved using the extragradient algorithm proposed in [2]. However, the structure of the auxiliary problem does not guarantee the fulfilment of the assumptions in [2] for the convergence of the numerical method. Thus, we show the convergence of the extragradient algorithm under different assumptions, without requiring that each feasible point is a fixed point for the constraint map. The algorithm's behaviour when applied to the auxiliary equilibrium problems is tested by preliminary numerical results.

References

(1) Aussel D., Sultana A. and V. Vetrivel (2016), On the existence of projected solutions of quasi-variational inequalities and generalized Nash equilibrium problems, *Journal of Optimization Theory and Applications*, vol.170, pp.818-837. ISSN 0022-3239. DOI: 10.1007/s10957-016-0951-9.

(2) Strodiot J.J., Nguyen T.T.V. and V.H. Nguyen (2013), A new class of hybrid extragradient algorithms for solving quasi-equilibrium problems, *Journal of Global Optimization*, vol.56, pp.373-397. ISSN 0925-5001. DOI: 10.1007/s10898-011-9814-y.

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A numerical approach to projected solution for a pay-as-bid electricity market model

S. Latini * [1] , **R. Cambini** [2] , **M. Castellani** [3]

Abstract

The concept of projected solution is derived from an analysis of a pay-as-bid model in a deregulated electricity market proposed in [1]. This model poses three primary challenges: a bilevel structure, the presence of projection, and the need to solve N problems simultaneously. Aim of this work is to develop a numerical method for solving it.

The bilevel structure and the presence of projection can be avoided by means of the KKT conditions. We obtain a single level equilibrium problem, which is a potential game and it is possible to solve a single optimization problem instead of N simultaneous minimization problems.

However, the complementarity conditions, paired with the KKT conditions and needed to manage the binding constraints, make the problem difficult from a computational point of view. In this light, following the results in [2], two approaches are proposed to manage complementarity conditions, the first one based on the use of suitable binary dummy variables and the latter one based on a transversality condition.

Keywords

Projected solution, Electricity market model, Numerical method

References

(1) Aussel D., Sultana A. and V. Vetrivel (2016), On the existence of projected solutions of quasi-variational inequalities and generalized Nash equilibrium problems, *Journal of Optimization Theory and Applications*, vol.170, pp.818-837. ISSN 0022-3239. DOI: 10.1007/s10957-016-0951-9.

(2) Cambini R. and R. Riccardi, Optimality conditions for differentiable linearly constrained pseudoconvex programs, submitted to *Decis. Econ. Finance.*

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Least cores and energy communities

Giancarlo Bigi * [1] , **Davide Fioriti** [2] , **Antonio Frangioni** [3]
Mauro Passacantando [4] **Davide Poli** [5]

Abstract

Prosumers are energy consumers who can also produce it. They can create communities to share energy through an aggregator and authorities often provide incentives. A model of an energy community is presented as a coalitional TU-game. Properties of its core and quasi cores are studied, providing characterizations as well as estimates for the least core. The latters may facilitate the computation of fair shares of the overall benefits through row generation and other techniques for LPs with exponentially many constraints.

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Wednesday, September 4

Relationships between Global and Local Monotonicity of Operators

Boris Mordukhovich [1] ,

Abstract

The talk is devoted to establishing relationships between global and local monotonicity, as well as their maximality versions, for single-valued and set-valued mappings between finite-dimensional and infinite-dimensional spaces. We first show that for single-valued operators with convex domains in locally convex topological spaces, their continuity ensures that their global monotonicity agrees with the local one around any point of the graph. This also holds for set-valued mappings defined on the real line under a certain connectedness condition. The situation is different for set-valued operators in multidimensional spaces as demonstrated by an example of locally monotone operator on the plane that is not globally monotone. Finally, we invoke coderivative criteria from variational analysis to characterize both global and local maximal monotonicity of set-valued operators in Hilbert spaces to verify the equivalence between these monotonicity properties under the closed-graph and global hypomonotonicity assumptions.

References

(1) P. D. Khanh, V. V. H. Khoa, J. E. Martinez-Legaz and B. S. Mordukhovich (2024), Relationships between global and local monotonicity of operators, submitted, arXiv:2403.20227.

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On the regularity of selections and on Minty points of generalized monotone set-valued maps

Nicolas Hadjisavvas * [1] , **Monica Bianchi** [2] , **Rita Pini** [3]

Abstract

Given a generalized monotone set-valued map $T : X \rightrightarrows X^*$ on a Banach space X, it is clear that any map $T_1 : X \rightrightarrows X^*$ such that $T_1(x) \subseteq \mathbb{R}_{++}T(x)$ for all $x \in X$, has the same kind of generalized monotonicity as T. In addition, variational inequalities have the same solutions with respect to T, T_1.

In this talk, we explore the following question: Can we select a single-valued operator T_1 as above, which is continuous in some sense? We show, under various continuity assumptions on T, that indeed, there is a continuous selection of $\mathbb{R}_{++}T$. For example, we show that for every quasimonotone set-valued map T satisfying the Aubin property around $(y, y^*) \in \text{gph}(T)$ with $y^* \neq 0$ there exist locally Lipschitz selections of $\mathbb{R}_{++}T \setminus \{0\}$. In the last part some notions of Minty points of T are introduced, and their relationship with zeros as well as effective zeros of T is discussed; a correlation is established between these concepts and the broader context of the generalized monotonicity of the map T.

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Sensitivity Analysis for Generalized Equations via Strict Proto-Differentiability

Hang Nguyen * [1] [2] , **Ebrahim Sarabi** [3] ,

Abstract

This talk discusses a generalized differentiability property called *strict proto-differentiability* of subgradient mappings of diverse classes of functions including polyhedral, $\mathcal{C}^2$-composable, and $\mathcal{C}^2$-partly smooth functions. We demonstrate that strict proto differentiability of the underlying mapping implies the equivalence between two important stability properties of solution mapping to a generalized equation, namely metric regularity and strong metric regularity. This provides with us a novel route to analyze differential sensitivity of solutions to generalized equations under canonical/general perturbations.

References

(1) Hang, N.T.V., Jung, W., and Sarabi, M.E. (2024), Role of subgradients in variational analysis of polyhedral functions, *Journal of Optimization Theory and Applications*, vol.200, n.3, pp.1160-1192. ISSN 0022-3239. DOI: 10.1007/s10957-024-02378-6.

(2) Hang, N.T.V. and Sarabi, M.E. (2024), A chain rule for strict twice epi-differentiability and its applications, *SIAM Journal in Optimization*, vol.34, n.1, pp.918-945. ISSN 1052-6234. DOI: 10.1137/22M1520025.

(3) Hang, N.T.V. and Sarabi, M.E. (2023), Smoothness of subgardient mappings and its applications in parametric optimization, ArXiv preprint: 2311.06026.

(4) Hang, N.T.V. and Sarabi, M.E. (2024), A fresh look into variational analysis of $\mathcal{C}^2$-partly smooth functions, *Working paper*.

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Other regarding preferences in a pure exchange economy: existence of equilibrium with a variational inequality approach

Maria Bernadette Donato * [1] , **Antonio Villanacci** [2]

Abstract

We consider a pure exchange economy where households are allowed to deliver commodity transfers to other households. More precisely, we introduce a general equilibrium model with voluntary transfers among agents, where each household's preferences depend on her own consumption and other households' wealth. For such a model, we provide an existence result by using a variational inequality approach. Firstly, a quasi-variational inequality reformulation of the non-convex model (where households' preferences are not assumed to be convex) is proposed and used to achieve the existence of solutions in a compact set. Subsequently, we prove that the solutions to the quasi-variational inequality problem are equilibrium solutions with an upper bound on the consumptions. Finally, we get the existence of the equilibrium without the compactness of the consumption set.

References

(1) Kranich, I.J. (1988), Altruism and efficiency: a welfare analysis of the Walrasian mechanism with transfer, *Journal of public economics*, vol 36, pp. 369–386. DOI: 10.1016/0047-2727(88)90016-3.

(2) Donato, M.B. and Villanacci, A. (2023), Variational inequalities, maximal elements and economic equilibria, *Journal of Mathematical Analysis and Applications*, vol. 519 n. 1, pp. 1-24. DOI: 10.1016/j.jmaa.2022.126769.

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Vector-Valued Games: Characterization of Equilibria in Matrix Games

Matteo Rocca * [1] , **Giovanni P. Crespi** [2] , **Daishi Kuroiwa** [3]

Abstract The study of equilibrium concepts for games with vector-valued payoffs has been introduced by Shapley in 1959 in the framework of zero-sum games. Later, the same issue has been investigated in Borm et al.(1988), where existence of Nash equilibria for matrix games with two players has been proved. For games with general vector-valued payoffs, the notion of Nash and weak Nash equilibrium has been investigated by Wang (1991), assuming the classical Pareto ordering. In the same framework, the notion of proper Nash equilibrium appeared in Zhao (1991). Further investigations on vector games can be found in the literature. Following this stream of research, in this paper we provide general notions of Nash equilibrium, weak Nash equilibrium and proper Nash equilibrium, together with existence results and application to the special case of vector-valued matrix games. In general abstract setting the relations among the three notions are also explored, providing a general scheme to characterize the set of equilibria. Multiobjective games provide a more realistic representation of decision-making situations where conflicting objectives are present, allowing for a richer analysis of strategic interactions among players. Examples arise in various fields of application, such as Merging and Acquisition; Environmental Resource Management; Supply Chain Management; Project Management; Energy Market; Water Resource Allocation and many more. This motivates our interest in deepening the study of equilibria for this type of games. Moreover, a special case, useful in applications, appears to be that of vector matrix games. For this setting, we show that the notions of proper Nash equilibrium and Nash equilibrium are equivalent and we prove existence of both Nash equilibria and weak Nash equilibria.
Further, we present an approach for describing the set of all Nash equilibria and weak Nash equilibria for a two person two strategies multiobjective matrix game. By means of this approach we compute the best correspondence maps for a general two players, two strategies, two objectives matrix game. We illustrate the obtained results by examples.

References

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(1) Shapley, L.S. (1959), Equilibrium Points in Games with Vector Payoffs. *Nav Res Logist Quart.*, vol. 6, pp. 57–61.

(2) Borm, P. E. M., Tijs, S. H., van den Aarssen, J. C. M. (1988), Pareto Equilibria in Multiobjective Games. *Methods of Operations Research.*, vol. 60, pp. 303–312.

(3) Wang, S.Y. (1991), An Existence Theorem of a Pareto Equilibrium. *Appl. Math. Lett.*, vol. 4, pp. 61–63.

(4) Zhao, J. (1991), The equilibria of a multiple objective game. *Int. J. Game.Theory*, vol. 20, pp. 171–182.

Jensen type inequalities for generalized (m, M, ψ)-convex functions with applications

Slavica Ivelić Bradanović * [1] , **Silvestru Sever Dragomir** [2] , **Neda Lovričević** [3]

Abstract

Among various generalized classes of convexity, the class of (m, M, ψ)-convex functions, introduced by Dragomir in 2001, has attracted increasing attention recently. This class covers many other subclasses of convexity, such as the class of strongly convex functions, delta convex functions, approximately concave functions and others. In this work, we present some new characterizations of this generalized class of convexity. We also prove new Jensen type inequalities for (m, M, ψ)-convex functions. Our results extend and improve the corresponding results in the literature valid for different subclasses of convex functions. As application of the main results, we derive new lower and upper bounds estimations for some well-known mean inequalities.

References

(1) S. S. Dragomir, Asurvey on Jessen's type inequalities for positive functionals. In: Pardalos, P. M., et al. (eds.) Nonlinear Analysis. Springer Optimmization and Its Applications 68, Springer, New York (2012) 177–232.

(2) S. S. Dragomir, S. Ivelić Bradanović and N. Lovričević, Jensen type inequalities for (m, M, ψ)-convex functions with applications, submitted.

(3) S. S.Dragomir, S.Ivelić Bradanović and N. Lovričević, Majorization type inequalities for (m, M, ψ)-convex functions with applications, submitted.

(4) S. S. Dragomir and K. Nikodem, Functions generating (m, M, ψ)-Schur-convex sums, Aequat. Math. 93 (2019), 79-90.

(5) S. Ivelić Bradanović, Improvements of Jensen's inequality and its converse for strongly convex functions with applications to strongly f-divergences, J. Math. Anal. Appl. 2 (531), (2024),1-16.

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Generalized Jensen and Jensen-Mercer inequalities for strongly convex functions with applications

Neda Lovričević * [1] , **Slavica Ivelić Bradanović** [2]

Abstract

Strongly convex functions as a subclass of convex functions, still equipped with stronger properties are employed through several generalizations and improvements of the Jensen inequality and the Jensen-Mercer inequality. In addition, applications of the main results are provided and given in the form of new estimates for so called strong f- divergences: the concept of the Csiszár f-divergences for strongly convex functions f, together with the special cases (Kullback-Leibler divergence, χ^2-divergence, Hellinger divergence, Bhattacharya distance, Jeffreys distance and Jensen-Shannon divergence). Furthermore, new estimates for the Shannon entropy are obtained and finally, new Chebyshev type inequalities are derived.

References

(1) S. Ivelić Bradanović, N. Lovričević, Generalized Jensen and Jensen-Mercer inequalities for strongly convex functions with applications, submitted.

(2) S. Ivelić Bradanović, Improvements of Jensen's inequality and its converse for strongly convex functions with applications to strong f-divergences, J. Math. Anal. Appl. 2 (531) (2024), 1-16.

(3) M. Klaričić Bakula, M. Matić, J. Pečarić, On some general inequalities related to Jensen's inequality, Internat. Ser. Numer. Math. 157 (2008), 233-243.

(4) M. Krnić, N. Lovričević, J. Pečarić, On some properties of Jensen-Mercer's functional, J. Math. Inequal. 6 (1) (2012), 125-139.

(5) N. Lovričević, D. Pečarić, J. Pečarić, Monotonicity of the Jensen functional for f-divergences via the Zipf-Mandelbrot law, Math. Inequal. Appl., 22 (2019) 4; 1427-1449.

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Thursday, September 5

Nonconvex quadratic optimization over two quadrics and the unit sphere

Fabián Flores-Bazán * [1] , **Felipe Opazo** [2]

Abstract

We consider the problem of minimizing a quadratic function over two quadrics and the unit sphere, and develop a geometric analysis via the study of the convexity of the Minkowski sum $K + C$ with C being any convex pointed cone, and K is a set such that the convex envelope of $K + C$ is closed. Besides the standard dual problem, two additional duals are associated to the original problem. Thus, various zero duality gaps and corresponding strong duality properties are introduced and then analyzed.

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Forward-backward algorithms devised by graphs

Francisco J. Aragón-Artacho * [1] , **Rubén Campoy** [2] , **César López-Pastor** [3]

Abstract

In this work, we present a methodology for devising forward-backward methods for finding zeros in the sum of a finite number of maximally monotone operators. We extend the techniques from (3) to cover the case involving a finite number of cocoercive operators, which should be directly evaluated by the algorithm instead of computing their resolvent. The algorithms are induced by three graphs that determine how the algorithm variables interact with each other and how they are combined to compute each resolvent. The hypotheses on these graphs ensure that the algorithms obtained have minimal lifting and are frugal, meaning that the ambient space of the underlying fixed point operator has minimal dimension and that each resolvent and each cocoercive operator is evaluated only once per iteration. This framework allows to recover some known methods, as well as generating new ones.

References

(1) Aragón-Artacho, F. J., Malitsky, Y., Tam, M. K. and Torregrosa-Belén, D. (2023), Distributed forward-backward methods for ring networks, *Computational Optimization and Applications*, vol.86, n.3, pp.845-870. ISSN 1573-2894. DOI: 10.1007/s10589-022-00400-z.

(2) Bredies, K., Chenchene, E. and Naldi, E. (2024), Graph and Distributed Extensions of the Douglas–Rachford Method, *SIAM Journal on Optimization*, vol.34, n.2, pp.1569-1594. ISSN 1095-7189. DOI: 10.1137/22M1535097.

(3) Bredies, K., Chenchene, E., Lorenz, D. A. and Naldi, E. (2022), Degenerate Preconditioned Proximal Point Algorithms, *SIAM Journal on Optimization*, vol.32, n.3, pp.2376-2401. ISSN 1095-7189. DOI: 10.1137/21M1448112.

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On nonmonotone extensions of Spingarn's method

Puya Latafat * [1] , **Brecht Evens** [2] , **Panagiotis Patrinos** [3]

Abstract

Spingarn's method of partial inverses and the progressive decoupling algorithm address inclusion problems involving the sum of an operator and the normal cone of linear subspace. This so-called linkage problem is crucial for developing numerical methods for many structured optimization problems. Existing convergence results have relied on either monotonicity or elicitable monotonicity assumptions. In this work, we introduce a new class of nonmonotone tractable problems that goes beyond the elicitable monotone setting. Our convergence result relies on a reformulation of relaxed Spingarn's method as an instance of a recently proposed nonmonotone preconditioned proximal point method. Additionally, we showcase range of problems our theory is able to cover through illustrative examples.

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Hidden Convexity in the ℓ_0 Pseudonorm and in Sparse Optimization

Michel De Lara * [1] , **Jean-Philippe Chancelier** [2]

Abstract

The so-called ℓ_0 pseudonorm counts the number of nonzero entries of a vector. As a function, it is piecewise constant. However, we show that the ℓ_0 pseudonorm displays hidden convexity and we discuss possible applications in sparse optimization. The talk is organized in three parts. First, we provide background on so-called couplings and Fenchel-Moreau conjugates. In generalized convexity, the duality product is replaced by a coupling c, the Fenchel conjugacy by the c-conjugacy associated with the coupling c, closed convex functions by c-convex functions (functions that are equal to their c-biconjugates), and subdifferentials by c-subdifferentials. Second, we review (surprising) results about the ℓ_0 pseudonorm. We present the E-Capra coupling, derived from the Euclidean norm, and we show that the ℓ_0 pseudonorm is E-Capra-convex. We deduce that ℓ_0 coincides, on the unit sphere, with a proper convex lsc function — that we call the ℓ_0-cup and that we describe. Then, we go beyond the Euclidean norm, and we outline the role of orthant-strictly monotonous norms. Third, we discuss possible applications in sparse optimization: Fermat rule, Capra-cuts method and sparsity-inducing norms.

References

(1) J.-P. Chancelier and M. De Lara, Hidden convexity in the ℓ_0 pseudonorm, *Journal of Convex Analysis*, 28(1):203–236, 2021.

(2) J.-P. Chancelier and M. De Lara, Capra-convexity, convex factorization and variational formulations for the ℓ_0 pseudonorm, *Set-Valued and Variational Analysis*, 30:597–619, 2022.

(3) J.-P. Chancelier and M. De Lara, Orthant-strictly monotonic norms, generalized top-k and k-support norms and the ℓ_0 pseudonorm, *Journal of Convex Analysis (to appear)*, 2022.

(4) A. Deza, J.-B. Hiriart-Urruty, and L. Pournin, Polytopal balls arising in optimization, *Contributions to Discrete Mathematics*, 16(3):125–138, 2021.

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Abstract Cutting Plane Method Applied to Sparse Optimization

Jean-Philippe Chancelier [1] , **Michel De Lara** [2] ,
Seta Rakotomandimby * [3]

Abstract In usual convexity, a function is closed convex if and only if it is the supremum of its affine minorants. In abstract convexity, affine functions are replaced by other elementary functions belonging to some set H. By definition, a function is abstract H-convex if it is the supremum of its H-minorants (3). Few abstract H-convex minimization algorithms have been studied, (3, Chapter 9), such as the abstract cutting plane method and the abstract branch-and-bound method. Recent work (1) on the ℓ_0 pseudonorm has highlighted elementary functions that makes the ℓ_0 pseudonorm abstract H-convex — the so-called E-CApra "affine" functions. As explicit formulas for the E-Capra subdifferential of the ℓ_0 pseudonorm have been calculated (2), it has been made possible to implement abstract H-convex minimization algorithms to the special case of the ℓ_0 pseudonorm.

We present our numerical tests on the E-CAPRA cutting plane method applied to the following sparse optimization problems: the minimization of the ℓ_0 pseudonorm in a blunt closed cone, and the computation of the spark of a matrix (known to be a NP-hard problem (4)).

References

(1) J.-P. Chancelier and M. De Lara. Capra-convexity, convex factorization and variational formulations for the ℓ_0 pseudonorm. *Set-Valued and Variational Analysis*, 30:597-619, 2022.

(2) A. Le Franc, J. P. Chancelier and M. De Lara. The Capra-subdifferential of the ℓ_0 pseudonorm. *Optimization*, 73(4), 1229-1251, 2024.

(3) A. Rubinov. *Abstract convexity and global optimization*, volume 44 of *Nonconvex Optimization and its Applications*. Kluwer Academic Publishers, Dordrecht, 2000.

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(4) A. M. Tillmann and M. E. Pfetsch. The computational complexity of the restricted isometry property, the nullspace property, and related concepts in compressed sensing. *IEEE Transactions on Information Theory*, 60(2):1248-1259, 2014.

Efficient solutions of uncertain multiobjective optimization problems

César Gutiérrez * [1]

Abstract

This talk addresses an uncertain multiobjective optimization problem. Namely, countably many scenarios are considered, which concern both the objective function and the ordering cone of the problem. In this setting, new notions of solution are introduced, which generalize the so-called efficient solutions (see Abdelaziz et al. (1994), Köbis (2015), Ide and Schöbel (2016) and the references therein). Some relationships between them and other concepts of solution of the literature are shown. Finally, necessary and sufficient optimality conditions are derived by linear scalarization in problems satisfying generalized convexity assumptions. These characterizations extend and clarify some results in Engau and Singler (2020).

References

(1) Abdelaziz, F.B., Lang, P., Nadeau, R. (1994), Pointwise efficiency in multiobjective stochastic linear programming, *Journal of the Operational Research Society*, vol.45, n.11, pp.1324-1334. ISSN 0160-5682. DOI: 10.2307/2583860.

(2) Engau, A., Sigler, D. (2020), Pareto solutions in multicriteria optimization under uncertainty, *European Journal of Operational Research*, vol.281, n.2, pp.357-368. ISSN 0377-2217. DOI: 10.1016/j.ejor.2019.08.040.

(3) Ide, J., Schöbel, A. (2016), Robustness for uncertain multi-objective optimization: a survey and analysis of different concepts, *OR Spectrum*, vol.38, pp.235-271. ISSN 0171-6468. DOI: 10.1007/s00291-015-0418-7.

(4) Köbis, E. (2015), On robust optimization: relations between scalar robust optimization and unconstrained multicriteria optimization, *Journal of Optimization Theory and Applications*, vol.167, pp.969-984. ISSN 0022-3239. DOI: 10.1007/s10957-013-0421-6.

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On Constraint Qualifications for Mathematical Programming Problems with Vanishing Constraints on Hadamard Manifolds

Balendu Bhooshan Upadhyay * [1]

Abstract

This talk is concerned about the study of mathematical programming problems with vanishing constraints on Hadamard manifolds (in short, MPVC-HM). We introduce the Abadie constraint qualification (in short, ACQ) and (MPVC-HM)-tailored ACQ for MPVC-HM and provide some necessary conditions for the satisfaction of ACQ for MPVC-HM. Moreover, we demonstrate that the Guignard constraint qualification (in short, GCQ) is satisfied for MPVC-HM under certain mild restrictions. We present several (MPVC-HM)-tailored constraint qualifications in the framework of Hadamard manifolds that ensure satisfaction of GCQ. Moreover, we refine our analysis and present some modified sufficient conditions which guarantee that GCQ is satisfied. Several non-trivial examples are incorporated to illustrate the significance of the discussed results.

References

(1) Achtziger, W., Kanzow, C.: Mathematical programs with vanishing constraints: optimality conditions and constraint qualifications. Math. Program. 114, 69–99 (2008).

(2) Hoheisel, T., Kanzow, C.: Stationary conditions for mathematical programs with vanishing constraints using weak constraint qualifications. J. Math. Anal. Appl. 337(1), 292–310 (2008).

(3) Maeda, T.: Constraint qualifications in multiobjective optimization problems: differentiable case. J. Optim. Theory Appl. 80(3), 483–500 (1994).

(4) Rapcsák, T.: Smooth Nonlinear Optimization in Rn. Springer, Berlin (2013).

(5) Udriste, C.: Convex Functions and Optimization Methods on Riemannian Manifolds. Springer, Berlin (2013).

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Computer Assisted Visualization of m-convex Hulls of Sets

Attila Gilányi * [1] , **Roy Quintero** [2] , **Lan Nhi To** [3]

Abstract
The concept of m-convexity was introduced as a generalization of convexity by Gheorghe Toader in his paper [3]. In this talk, extending and generalizing the results proved in the papers [1] and [2], as well as in the talk entitled 'Computer assisted investigations connected to m-convexity of sets' presented at the 13th International Symposium on Generalized Convexity and Monotonicity in 2022, we investigate geometrical properties of m-convex sets. As a main results, we present a computer program developed in the computer algebra system Maple (a trademark of Waterloo Maple Inc.), which determines and visualizes m-convex hulls of finite sets of points in the two-dimensional plane and in the three-dimensional space.

References

(1) A. Gilányi, N. Merentes, and R. Quintero, *Mathability and an animation related to a convex-like property*, 7^{th} IEEE Conference on Cognitive Infocommunications (CogInfoCom), 227–232, IEEE, 2016.

(2) A. Gilányi, N. Merentes, and R. Quintero, *Presentation of an animation of the m-convex hull of sets* 7^{th} IEEE Conference on Cognitive Infocommunications (CogInfoCom),307–308, IEEE, 2016.

(3) G. Toader, *Some generalizations of the convexity*, Proc. Coll. Approx. and Opt. (Cluj-Napoca), 329–338, 1984.

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SEPTEMBER 2-6 2024 UNIVERSITY OF PISA, ITALY
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Google PageRank, Citation Networks, and Optimization

Vu Thi Huong[1][2]

Abstract

Google's search engine was co-founded by two computer scientists, L. Page and S. Brin, while they were both PhD students at Stanford University in 1998. Since then, Google has become an indispensable part of our modern lives, providing a huge source of human knowledge. Each time we search, Google returns a list of millions of web pages in less than a second. Normally, we follow the links at the top of the list and find that those are the most relevant. The algorithm behind this is PageRank, a link analysis method to rank web pages based on their "importance". In this talk, we will discuss some mathematical aspects of PageRank, its application in academic citation networks, and how it is viewed through the lens of convex optimization. This allows us to further discuss whether the Polyak step rule in the gradient descent method is an optimal choice, from both theoretical and computational perspectives.

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A variance reduced stochastic conjugate gradient approach in smooth non-convex regime

Mahmoud M. Yahaya * [1] , **Poom Kumam** [2] , **Wiyada Kumam** [3]

Abstract

Stochastic gradient descent, SGD is the powerhouse and most used method to train Large-Scale Machine/Deep Learning models, partly due to the method's efficient use of the problem's structure. In this presentation, we are interested in stochastic conjugate gradient, the SCG descent method that often converges faster than SGD, unlike the existing approaches of SCG, which often relies on some strong assumptions such as boundedness supposition on the method's parameter known as conjugate gradient parameter. The introduced method has fewer assumptons and is devised based on the biased stochastic variance reduced method, namely StochAstic RecurSive grAdient algoritHm, SARAH; thus, this biased gradient estimator is incorporated into a newly formulated stochastic conjugate gradient direction. Moreover, an adaptive learning rate that is efficient and tune-free is also introduced, thus avoiding the usual manual tuning of the learning-rate strategy that is mostly adopted by SGDs. The proposed method is named SVRCG-ADA, which is simple to implement and performs efficiently for a wide variety of large-scale learning problems. Furthermore, we extensively analyze the convergence and complexity guarantees for convex and non-convex classes of smooth problems. Finally, we experimentally discuss the computational performance of SVRCG-ADA on large-scale real-world datasets through some learning problems and compare it with other state-of-the-art existing methods. The result suggested a superior performance of SVRCG-ADA on most of the considered problems.

References

(1) Herbert Robbins and Sutton Monro. "A stochastic approximation method", Annals of Mathematical Statistics, 22(3):400(407), 1951.

(2) W. W. Hager and H. Zhang, "A survey of nonlinear conjugate gradient methods", Pacific journal of Optimization, vol. 2, no. 1, pp. 35–58, 2006.

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Trajectory Controllability of Dynamical System Involving a Generalized Fractional Caputo's Derivative

Ganga Ram Gautam * [1]

Abstract

This article deals with the trajectory controllability (a stronger concept of exact controllability) of dynamical system conferred by generalized fractional order Caputo's derivative. Using generalized Laplace transform a equivalent integral equation is derived and with the help of fixed point technique trajectory controllability is proved of a considered system. Finally, an application is presented, in the form of an example of a recently introduced chaotic financial system with generalized fractional derivative to support our theoretical results. After that, the considered system is analyzed and discussed for its chaotic behavior under trajectory controllability.

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Existence and properties of inverse Muth probability model

Mukesh Kumar * [1] , **Agni Saroj** [2]

Abstract

The importance of upside down bathtub (UBT) shaped failure rate has various application in real life was discussed in literature. Which indicates that the inverted versions of usual distributions are capable of modeling the data with UBT-shaped failure rate. The proposed inverse Muth probability model is estimated using classical and Bayesian methods with the application of head and neck cancer data.

References

(1) Saroj, A., Sonker, P. K., & Kumar, M. (2022). Statistical properties and application of a transformed lifetime distribution: inverse muth distribution. Reliability: Theory & Applications, 17(1 (67)), 178-193.

(2) Chesneau, C., & Agiwal, V. (2021). Statistical theory and practice of the inverse power Muth distribution. Journal of Computational Mathematics and Data Science, 1, 100004.

(3) Jodra, P., Jimenez-Gamero, M. D., & Alba-Fernandez, M. V. (2015). On the Muth distribution. Mathematical Modelling and Analysis, 20(3), 291-310.

(4) Maya, R., Jodra, P., Irshad, M. R., & Krishna, A. (2022). The unit Muth distribution: statistical properties and applications. Ricerche di Matematica, 1-24.

(5) Barrios-Blanco, L., Gallardo, D. I., Gómez, H. J., & Bourguignon, M. (2023). A Compound Class of Inverse-Power Muth and Power Series Distributions. Axioms, 12(4), 383.

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Friday, September 6

SEPTEMBER 2-6 2024 UNIVERSITY OF PISA, ITALY
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Near Convexity and Generalized Differentiation

Nguyen Mau Nam [1] , **Nguyen Nang Thieu** * [2] , **Nguyen Dong Yen** [3]

Abstract

In this paper, we introduce the concept of nearly convex set-valued mappings and investigate fundamental properties of these mappings. Additionally, we establish a geometric approach for generalized differentiation of nearly convex set-valued mappings and nearly convex functions. Our contributions expand the current knowledge of nearly convex sets and functions, while providing several new results involving nearly convex set-valued mappings.

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Convex structure induced by geodesic convexity

Mihály Bessenyei * [1] ,

Abstract

In our talk, we discuss the connection of the convex geometric and the analytic properties of the underlying manifold. As an application of the main results we show that in Riemannian manifolds of dimension at least three, local versions the Peano property, the drop completeness, or the "thinness" of triangles are equivalent with having constant curvature, and global versions characterize Euclidean and hyperbolic spaces among Cartan–Hadamard manifolds.

References

(1) Bessenyei, M., Kertész, D. Cs., and Lovas, R. L. (2021), A sandwich with segment convexity, *J. Math. Anal. Appl.*, **500** art. 125108.

(2) Bessenyei, M., Kertész, D. Cs. (2023), Peano property, two dimensional triangles, constant curvature, *J. Math. Anal. Appl.*, **526** art. 127205.

(3) Lytchak, A., Petrunin, A. (2022), About every convex set in any generic Riemannian manifold, *J. Reine Angew. Math.*, **782** 235–245.

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Some results on non-symmetrically convex sets

Tibor Kiss * [1]

Abstract

As is widely known, the convex hull of the union of a convex subset and a point of a linear space equals to the union of the segments starting at the given point and ending in the set in question. This result is called the *drop theorem*. In the talk we will restrict ourselves to the real number line and deal with a variant of this result.

For a fixed parameter $t \in [0, 1]$, we say that a subset $D \subseteq \Re$ is *non-symmetrically t-convex* if $tx + (1 - t)y \in D$ whenever $x, y \in D$ with $y \leq x$. To avoid the trivial cases, we also assume that $t \notin \{0, \frac{1}{2}, 1\}$.

In the talk we give a sufficient condition under which the non-symmetric t-convex hull of a non-symmetric t-convex segment and a point outside it can be represented in the way detailed above.

References

(1) M. Lewicki, A. Olbryś (2014), On non-symmetric t-convex functions, *Math. Inequal. Appl.* 17, no.1, pp. 95–100.

(2) K. Nikodem, Zs. Páles (2008) Note on t-quasiaffine functions, *Ann. Univ. Sci. Budapest. Sect. Comput.* 29, pp. 127–139.

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Filippov Theorems for Differential Inclusions with Strengthened One-Sided Lipschitz Right-Hand Side

Robert Baier * [1] , **Elza Farkhi** [2] ,

Abstract

Set-valued maps which satisfy the *one-sided Lipschitz condition (OSL)* are first studied in Banach spaces in [(4)]. A simplified definition for multimaps in $\mathbb{R}^n$ and in Hilbert spaces is given in [(5)] and demands

$$\forall\, x, y \in \mathbb{R}^n,\ \forall v \in F(x)\ \exists w \in F(y):\ \langle x - y, v - w\rangle \le \mu \|x - y\|_2^2\,.$$

It is important here that the first selection of $F(x)$ is arbitrary while the second selection of $F(y)$ is chosen to satisfy the squared norm estimate. This set-valued generalization of monotonocity includes monotone decreasing functions, Lipschitz set-valued maps, and the pointwise negative of monotone set-valued maps, i.e. dissipative maps and allows for negative OSL constants μ. The weakening of the (set-valued) Lipschitz condition allows to study subclasses of right-hand sides being discontinuous in the state and still to recover convergence order at least $\mathcal{O}(\sqrt{h})$ with respect to the step size h of a set-valued Euler's method.

In applications in the area of control problems or differential inclusions the right-hand side even fulfills a strengthened version based on coordinate estimates, the so-called *strengthened OSL condition* introduced in [(7)] to improve the convergence order for Euler's method to 1. Some examples of OSL and SOSL right-hand sides will demonstrate the differences in the variants of (set-valued) Lipschitz continuity [(1)]. In the talk we focus on inner (and outer) perturbations of systems in continuous and discrete time. A piecewise linear interpolant of a discrete solution is a perturbed solution of the differential inclusion and the grid values of an absolutely continuous solution form a perturbed discrete solution of the difference inclusion. Filippov theorems for systems in continuous *and* discrete time with inner perturbations as in [(6)], [(2)] help to structure convergence proofs.

The talk is based on a collaboration with Elza Farkhi and on results in [(2)]–[(3)].

References

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(1) Baier R., and E. Farkhi (2013), Regularity of set-valued maps and their selections through set differences. Part 2: One-sided Lipschitz properties, *in: Serdica Mathematical Journal*, vol.39, no.3–4, pp.391–422. ISSN 1310-6600. URL: http://www.math.bas.bg/serdica/n34_13.html

(2) Baier R., and E. Farkhi (2018), Discrete Filippov-type stability for one-sided Lipschitzian difference inclusions, *in: Feichtinger G., Kovacevic R. and G. Tragler (eds.), Control Systems and Mathematical Methods in Economics, Lecture Notes in Economics and Mathematical Systems*, vol.687, pp.27–55. ISSN 0075-8442. DOI: 10.1007/978-3-319-75169-6_3.

(3) Baier R., and E. Farkhi (2023), A Filippov approximation theorem for strengthened one-sided Lipschitz differential inclusions, *Computational Optimization and Applications*, vol.86, n.3, pp.885–923. ISSN 0926-6003. DOI: 10.1007/s10589-023-00517-9.

(4) Donchev T.D. (1991), Functional-differential inclusion with monotone right-hand side, *Nonlinear Analysis. Theory, Methods & Applications*, vol.16, n.6, pp.533–542. ISSN 0362-546X. DOI: 10.1016/0362-546X(91)90026-W.

(5) Donchev T.D., and E. Farkhi (1998), Stability and Euler approximation of one-sided Lipschitz differential inclusions, *SIAM Journal on Control and Optimization*, vol.36, n.2, pp.780–796. ISSN 0363-0129. DOI: 10.1137/S0363012995293694.

(6) Donchev T.D., and E. Farkhi (2009), On the theorem of Filippov-Pliś and some applications, *Control and Cybernetics*, vol.38, n.4A, pp.1251–1271. ISSN 0324-8569. URL: http://control.ibspan.waw.pl:3000/contents/show/17?year=2009

(7) Lempio F. and V.M. Veliov (1998), Discrete approximations of differential inclusions, *Bayreuther Mathematische Schriften*, vol.54, pp.149–232. ISSN 0172-1062. URL: https://www.math.uni-bayreuth.de/en/research/publications/bms/

On generalized vector quasi-equilibrium problems

Adela Elisabeta Capătă * [1]

Abstract

In this talk, I will present some existence results for solutions to a generalized vector quasi-equilibrium problem with set-valued mappings and moving cones. The key of this approach is a Browder-type fixed point theorem [1], and it permits working in a new direction with the milder condition of transfer open-valued mapping and with the novel concept of upper $C(x)$-convex mapping. Within these results, $C(x)$ is assumed to be a pointed convex cone, and, no closedness assumptions are made on $C(x)$ and on $(-\text{int}C(x))^c$. The obtained results also provide sufficient conditions for the existence of solutions to other generalized vector quasi-equilibrium problems. By means of Horvath's continuous selection theorem, existence theorems are given for generalized quasi-equilibrium problems with trifunctions.

References

(1) Liu, J., Wang, M., Yuan, Y. (2020) Browder type fixed point theorems and Nash equilibria in generalized games. *Journal of Fixed Point Theory and Applications*, vol. 22, 71. ISSN 1661-7746. DOI: 10.1007/s11784-020-00806-4.

(2) Capătă, A.E. (2024), Generalized vector quasi-equilibrium problems, *Mathematics*, vol. 12, 809. ISSN 2227-7390. DOI: 10.3390/math12060809.

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Radial reversible betweenness spaces and BP-isomorphisms

**Janusz Morawiec * [1] , Martin Doležal [2] ,
Jan Kolář [2] , Oleksandr Maslyuchenko [1]**

Abstract A convex structure is called a *betweenness space* if its convexity is induced by the family of convex hulls of two-element sets (see Proposition 4.1.1 in (1)). Given a betweenness space X and $a, b \in X$ we denote by $[a, b]$ the convex hull of the set $\{a, b\}$ and put $(a, b) = [a, b] \setminus \{a, b\}$. If X and Y are betweenness spaces, then a map $f\colon X \to Y$ is said to be a *betweenness preserving function* (BP-function for short) if $f([a, b]) \subset [f(a), f(b)]$ for all $a, b \in X$, and a bijection $f\colon X \to Y$ is said to be a *betweenness preserving isomorphism* (BP-isomorphism) if f and f^{-1} are BP-functions.

Given a betweenness space X we define the *pseudointerior* and the set of *extreme points* of X by $I_X = \bigcup_{x,z \in X}(x, z)$ and $E_X = X \setminus I_X$, respectively. We say that a betweenness space X is *radial* if for any distinct points $x, y, z \in X$ we have $(x, y) \cap (x, z) \neq \emptyset \Longrightarrow z \in (x, y)$ or $y \in (x, z)$. We say that a betweenness space X is *reversible* if for all $a \in I_X$ and $x \in E_X$, there exists $z \in E_X$ such that $a \in (x, z)$; the point $z \in E_X$ is unique if the reversible betweenness space is also radial.

This talk is based on paper (2) and manuscript (3). Its aim is to give a characterization of BP-isomorphic radial reversible betweenness spaces and then provide an algebraic description of betweenness isomorphic classes in the family of all circles with three collinear points added inside.

References

(1) van de Vel, M.L.J. (2024), Theory of convex structures, *North-Holland Math. Library*, vol.50, xvi+540 pp. ISBN 0-444-81505-8.

(2) Doležal M., Kolář J. and Morawiec J. (2024), Betweenness isomorphisms in the plane – the case of a circle and points, *Advances in Geometry.*

(3) Doležal M., Kolář J., Maslyuchenko O., and Morawiec J. (2024), Extreme points of betweenness spaces and convex preserving isomorphisms, *Manuscript.*

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Approximation Algorithms in Computer Vision: Codes MATLAB

Mohamed Dalah * [1] , **Ammar Derbazi** [2]

Abstract

In this work, we give an approximation algorithm in computer vision. In first step, we write our algorithm with all conditions and we give some codes in Matlab Language. Finally, we give two numerical tests.

References

(1) Boykov, O. Veksler, and R. Zabih (2001), Fast approximate energy minimization via graph cuts, *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol.23, n.11, pp.1222-1239.

(2) G. H. Golub and C. F. Van Loan (1996), Matrix Computations, *Johns Hopkins Studies in Mathematical Sciences*.

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